Understanding the Complex Nature of Piano Tone

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Refereed

Abstract

The piano is among the foremost instruments used in classical music. It produces sound by striking metal strings with felt covered hammers. At first glance this system seems to be well described by the laws of transversely vibrating strings, but there are many subtle effects that change the sound produced from what this simple model predicts. Of these effects, the most important is the nonlinear interaction between the hammer and the string. These complications seem to account for a large part of the charm of the piano sound.

Inharmonicity and wrapped strings

The natural frequency of the nth mode in a transversely vibrating string is given by the formula ,

$$f_n = nf_1 = \frac{n}{2l}\sqrt{\frac{\tau}{\rho A}}$$

where n = 1, 2, 3..., l is the string length, τ is the tension, A is the string cross sectional area and ρ is the string density. However, real strings have nonzero bending stiffness, which leads to a nonlinear effect known as inharmonicity. The frequency of each mode is raised by a small amount, so that

$$f_n = nf_1\sqrt{1+Bn^2}$$

B is a factor determined by the material and dimensions of the string, such that

$$B = \frac{\pi^3 E d^4}{64 l^2 \tau} \qquad [1]$$

Here d is diameter, and E is Young's modulus for the string.

This explains the great string length of a typical grand piano: the extreme bass strings must be around 2 m long to achieve the low frequencies required (the lowest note on a standard piano has a fundamental of 27.5 Hz) without excessive inharmonicity, which gives a metallic sound.

The tension must be kept above a certain level to give effective coupling of the string to the bridge, and it is not possible to increase the unwound string diameter without raising the inharmonicity. The bass strings are wrapped with copper wire because this is an effective way of increasing the string mass without increasing the stiffness.

Inharmonic vibrations are dispersive, so different harmonics have different velocities, and the phase of each harmonic is constantly changing relative to the others.

It has long been supposed that the human ear is insensitive to phase [2], but recent experiments have shown that in the bass register, the arrival times of different harmonics can have a marked effect on the perceived sound [3].

In experiments [4] it was found that synthesized notes with zero inharmonicity are easily recognised as such, and judged as sounding dull. Notes where inharmonicity is too high are judged as sounding metallic, as might be heard from a 'baby' grand or an upright. It seems that the ear expects a certain amount of inharmonicity, which roughly corresponds to the dimensions of strings used in a full scale grand. Notes in this range are described as 'warm'.

The question of whether this is a response that is learned or one that is innate is one that is both interesting and beyond this author.

As a consequence of inharmonicity, piano tuners will deviate from equal temperament, sharpening the high treble notes and flattening the bass. The degree to which this is done is a matter of both personal taste and the intended use of the piano.

A piano intended for quiet living room playing may be tuned close to equal temperament, while a piano in a showroom may be tuned with a larger stretch, so that loud, brassy chords will sound better.

Miller [5] proposed a string weighting system (something like a blob of gum at one end of the string) which would reduce the inharmonicity, this could perhaps be a way of improving the sound of pianos with short strings such as uprights. However no further work seems to have been done on this topic.

Longitudinal string modes

Conklin [6, 7] has investigated the importance of longitudinal string modes, in which the string is periodically compressed then stretched lengthwise, as in the high school physics demonstration with a 'slinky'.

Although the lowest frequency longitudinal mode is always at least three octaves above the fundamental of transverse vibration and the amplitude is generally quite low, it was found that this mode can have a large effect on the piano tone, especially in the bass register. Higher longitudinal modes occur but their levels are lower still. The frequency of the lowest longitudinal mode is given by

$$f_l = \frac{1}{2l}\sqrt{\frac{E}{\rho\alpha}}$$

where α is the Rayleigh correction for lateral inertia [8], hence it is independent of the transverse modes. It may be excited by the action of the hammer (see the section on hammer excitation) or by coupling to a transverse mode. It is normally ignored by piano makers and tuners, but Conklin found that when this mode is in tune with the transverse modes the effect is pleasing, and if it is not in tune the effect is dissonant.

Several interesting sound examples are available online [9], in which melodies are played on strings with the same transverse modes but longitudinal modes tuned to semitones.

Decay rate

When the string vibrates, the bridge is forced up and down, in turn exciting the soundboard, which radiates acoustic energy. The simple model assumes that vibrations in the string decay exponentially as energy is removed. However the real string decay appears to be much more complex, and to depend on whether the note is in the treble or bass register.

From the moment the string is initially struck by the hammer, vibrations decay quickly and in an exponential manner for a short period of time, then change abruptly to give a much slower decay. This is known as compound decay, as can be seen in figure 1.

The effect on the listener is a kind of perceptual trick, where the note appears to be both loud and sustained, which should not be possible [10]. Compound decay is caused by two things, the first being the polarisation of string vibrations and second the use of 2, 3 or 4 strings for a single note and the deliberate mistuning between them.

The initial vibration of the string is in the direction of the hammer motion, perpendicular to the soundboard (vertical polarisation). The sound board is a large plate, and is relatively compliant to out of plane motion. Over time (about 100 ms) the string vibration

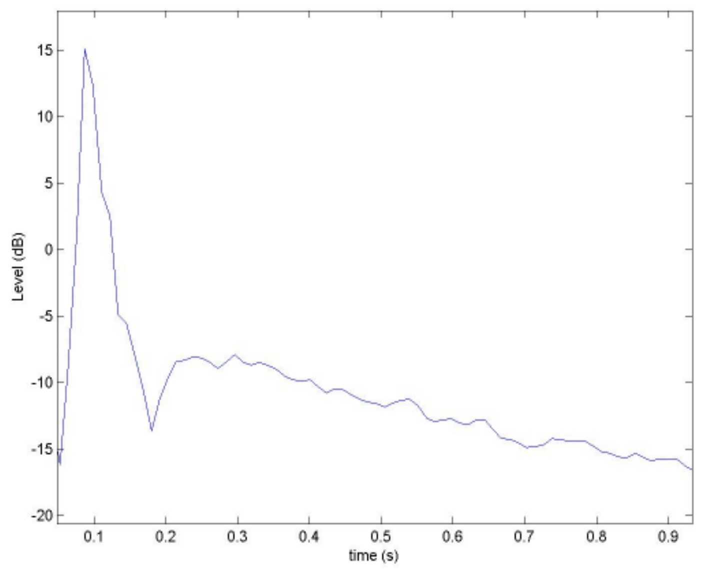

Figure 1: Measured level of piano note showing compound decay

changes polarisation so that it is moving in the same plane as the soundboard (horizontal polarisation).

The soundboard is much stiffer in this direction, and so energy is transferred from the string at a much lower rate. The string continues to change between these two polarisations for the duration of the note.

Interestingly the Stuart piano [11], designed in Australia, uses a different string path over the bridge, which causes the strings to remain vertically polarised. Although only a few have been built so far, they have been praised for their 'singing tone' and 'stronger, more harmonious sound'.

The use of multiple strings also helps to give a compound decay [10]. Two or more strings are typically used for all notes except for the lowest octave, and they will be deliberately mistuned by around 1 Hz.

When the strings are struck, they are initially in phase, and so both will be forcing the bridge up or down at the same time. But as the note decays, the phase relationship changes (there is beating between the notes), and they are no longer working together to move the bridge.

Effectively the bridge impedance increases when the strings are out of phase, and the rate of energy transfer is much lower.

String excitation by the hammer

Many researchers have tried to examine the interaction between the hammer and the string. Although it is now a little out of date, Hall [12-17] offers an extensive review of the history of the theoretical, numerical and experimental treatment of the problem up to 1992.

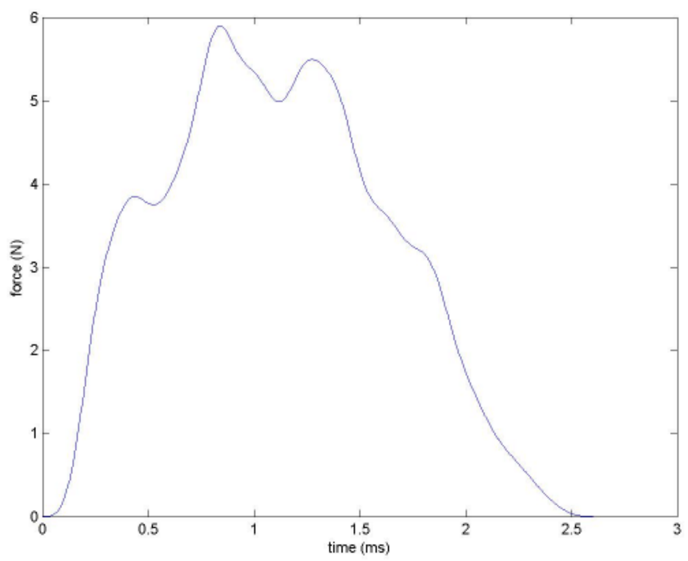

Figure 2: Simulated force history for a nonlinear, hysteretic hammer with typical properties

It is the nonlinear stiffness of the hammer which allows the player to vary the timbre created, through varying the hammer velocity. With a higher velocity, and therefore dynamic level, the hammer in striking the string is effectively harder, and the spectrum of the note contains more high frequencies than at lower dynamic levels.

Askenfelt and Jansson [18] call this "an automatic treble control connected to the volume knob", and it seems to be a characteristic of all acoustic instruments, and one that is essential for a 'natural' sounding instrument. Synthesizers that change dynamic levels simply by amplifying a note with a single, fixed spectrum sound unnatural.

The process of testing and changing the stiffness of individual hammers is known as voicing, and is one of the most important jobs of the piano tuner. Much like the stretch tuning, the required stiffness will change depending on the setting and the type of music to be played.

The hammer and string interaction proceeds like this: the hammer is thrown into the string by the piano action (driven by the player), slowing a little with the contact. Pulses from the impact spread in both directions from the hammer, and are reflected from the ends of the string. The arrival of reflected pulses decelerates the hammer and causes more secondary pulses.

With the arrival of more reflections, the hammer is thrown clear of the string, after which time it may regain contact. Finally there is no more interaction and the string is free to vibrate. The entire process takes between 0.5 ms for loud notes in the treble and 5 ms for quiet notes in the bass.

The number of reflected pulses and recontacts depends in general on the mass of the string and hammer mass, stiffness, velocity and contact point. In addition, more recent results [19] have shown that the hammer is both nonlinear and hysteretic. The hammer model presented by Stulov gives the force exerted on the string as a function of the current felt compression and the past compression history

$$F(t) = k\left[x(t)^p - \varepsilon \int_0^t x(t-\zeta)^p e^{\frac{-\zeta}{\tau_0}} d\zeta \right]$$

[19]

where k and ε are constants, x is the felt compression, p is the nonlinearity constant (p = 1 gives linear behaviour) and τ_0 is a time constant affecting the hysteresis.

With the force history calculated from a given hammer model as in figure 2, it is possible to simulate piano tones for evaluation. Examples created using the Stulov model can be found online [20], and as the reader will find out from listening there is no need to worry that the entire problem of piano modelling has already been solved!

There is some debate as to whether the only variable available to the pianist is the velocity of the hammer at contact, or if the pianist is able to control the timbre more closely through altering touch. It has been supposed [18] that the modes of vibration of the hammer and shank can play a role, by lengthening or shortening the duration of the hammer and string contact, changing the distribution of energy amongst the string harmonics.

Another possibility is one of the modes causing a rubbing motion of the hammer on the string that would excite longitudinal modes.

"...current computer simulations remain inadequate..."

Although these effects are probably small, professional pianists' assertions about the control they have over timbre suggests that they are important.

'Thump' noise

A major component of the sound of a piano is the noise of the action (the 'knock' or 'thump' noise) which accompanies the onset of each note. This is caused by various pieces of the action, particularly the key, reaching the end of their travel and creating a broadband vibration which is transmitted through to the soundboard and radiated (other parts also radiate but the soundboard is by far the most efficient).

This noise is particularly strong at around 90 Hz, which is typically where the fundamental soundboard mode is found. It is generally considered an integral part of the piano tone, as synthesizers which lack this component sound unnatural. However, an experiment in the Michael Fowler Centre [21] has shown that under certain circumstances, the knock noise can be too strong, leading to an unpleasant piano tone.

Conclusions

Piano sound production factors have been presented that are necessary for creating a warm and natural tone. A most important feature, allowing the player close control over the timbre, is the nonlinear stiffness of the hammer felt.

Other important components include inharmonicity, longitudinal string modes, compound decay rate and thump noise, and physical mechanisms have been given for each.

Listening to online examples shows that current computer simulations remain inadequate, so further work is needed to achieve a satisfactory model of the piano.

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